ERCIM "ALAIN BENSOUSSAN"
FELLOWSHIP PROGRAMME

ERCIM
European Research Consortium
for Informatics and Mathematics

Scientific Report

First name / Family name	Marta de León Contreras
Nationality	Spain
Name of the *Host Organisation*	NTNU
First Name / family name of the *Scientific Coordinator*	Kristian Seip and Karl-Mikael Perfekt
Period of the fellowship	01/09/2021 to 31/08/2022

I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

During the fellowship the fellow worked mainly with K.M. Perfekt. They have been studying the well-posedness of transmission problems between a dielectric material and a metamaterial, set in an open bounded subset of $\mathbb{R}^3$. That is, consider a domain $\Omega \subseteq \mathbb{R}^n$ that contains two domains Ω_1 and Ω_2, such that $\overline{\Omega} = \overline{\Omega_1} \cup \overline{\Omega_2}$ and $\Omega_1 \cap \Omega_2 = \emptyset$, $\partial\Omega_1$ has singularities, and there is a bounded function σ on Ω that takes strictly positive values in Ω_1 and strictly negative in Ω_2. Thus, we want to study the well-posedness of

$$\begin{cases} -div(\sigma\nabla u) = f\,, & in\ \Omega \\ u = 0, & in\ \partial\Omega \end{cases}'$$

in Sobolev spaces.

Many authors have recently worked in this kind of problems by using different approaches, like PDE and numerical analysis techniques. We have studied this problem in the case where Ω_1 is a cone and Ω is the unit ball by using an operator spectral theory point of view.

We have been able to establish the relationship between the approach using PDE techniques and ours so that now we can translate the properties for the operators involved that were proved with one approach into the other setting and vice-versa.

One of the researchers that have studied these problems is professor L. Chesnel, who is a researcher at INRIA, and collaborators. For that reason, the fellow visited him last May as part of the research training program (REP), in order to share results and interchange ideas. From this visit we have started a collaboration between both team works that we expect to obtain promising results. In particular, his former PhD student has joined us to our project in order to see if we can extend our results to more general domains or if, thanks to the techniques that we have used in the paper we can get a more direct and easy to follow proof of a known result about the compactness of the Neumann-Poincaré operator in C^1 interfaces.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

During the fellowship the fellow has published 2 papers in collaboration with people from Spain and Kazakhstan in Q1 papers.

1. *Variation operators for semigroups associated with Fourier-Bessel expansions.* with J. J. Betancor and A. J. Castro. COMMUNICATIONS ON PURE AND APPLIED ANALYSIS. Volume 21, Number 1, January 2022 pp. 239–273. doi:10.3934/cpaa.2021176

2. *he Hardy-Littlewood property and maximal operators associated with the inverse Gauss measure, with J. J. Betancor and A.J. Castro. Ann. Sc. Norm. Super. Pisa Cl. Sci. (2021) DOI: 10.2422/2036 2145.202011_020.*

Also, the fellow is finishing a paper with K. M. Perfekt and M. Rihani that we expect to submit to a journal in the following weeks.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

The fellow has attended to the analysis seminars and colloquiums of the department at NTNY during the year. In particular, she delivered one seminar talk in November 2021. Also, she gave a talk in the harmonic analysis seminar of the university of Würzburg, in Germany.

Moreover, the fellow has attended to 2 conferences last June. One, celebrated in El Escorial (Madrid, Spain), where she gave a talk, and the other at NTNU, in honor to Kristian Seip and Eero Saksman.

IV – RESEARCH EXCHANGE PROGRAMME (REP)

The fellow was visiting professor Lucas Chesnel in May 15-21 at Inria, in Paris.
She was interchanging ideas with the teamwork of Lucas and we are now involved in 2 projects and working together.
Professor Chesnel and his group have worked in the solvability of the problem mentioned in epigraph I in different domains with singularities and they have got their results by using mainly the T-coercivity approach, a PDE tool. The fellow has been working with K.M. Perfekt in this topic by using spectral theory techniques and she has shared with Lucas and his team the results that they have been able to obtain and the relationship between both approaches.